Sets ,Relations and Functions

Quantitative Aptitude & Business Statistics

Sets

- A collection of object (with the same type)
  - {1,2,3,...}
  - {1/2,2/3,3/4,...}
  - {Pen, Pencil, Book}

- Defining a set:
  - By listing (enumerating) the elements (for finite sets)
    - {Pen, Pencil, Book}
  - By explicit description of the elements (usually for infinite sets)
    - $\{X/Y \mid X, Y \in N, X=Y-1\}$

- Subset: R is subset of S if every element of R are coming also element in S.
  - R={2,3}
  - S={1,2,3}

- Power set: P is the power set of S (denoted as 2^S) if it includes all subsets of S.
- P={{}, {1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}

- **Null Set (or Empty set or void set) :A set having no element is called an empty set or void set .It is denoted by ϕ or { }**
- **Example :**
- **A={x: x is an even number not divisible by 2}**
- **B={x: x is a real number x^2=-1}**

- Singleton Set: **A set having only one element is called a singleton set.**
- **Example : A={ x: x is prime minister of India}**
- **B={2}**
- Pair Set : **A set having two elements is called a pair set.**
- **Example:{(1,2) ,(0,3),(4,9)} etc**

- **Finite Set:** A set having a finite number of elements i. e a set ,where counting elements is possible is called as a finite set.
- Examples: A={1,2,4,6} is a finite set because it has four elements

- **Infinite Set:** A set having a infinite number of elements i. e a set ,where counting elements is impossible is called as an infinite set.
- Examples : A={x: x is a set of all natural numbers }
- B={set of all points on the arc of a circle}

- **Equal Sets:** Two Sets A and B are said be equal ,if every element of A is in B and every element of b is in A and we write A≠B
- **Example;** The elements of a set may be listed in any order thus ,{1,2,3}={2,3,1} etc
- Equivalent sets : Two finite sets A and B are said to be equivalent if they have the same number of elements .we write A~B

- **Universal set:** **Any set which is super set of all the sets under consideration is known as the universal set and is either denoted by Ω or S or U**
- **For Example Let A ={1,2,3} ,B={3,4,5,6}**

  and C={0,1}
- **We take S={0,1,2,3,4,5,6,7,8,9}**

Laws of Operations

- 1.i) $A \cup A = A$ ii) $A \cup \phi = A$ iii) $A \cap A = A$

  iv) $A \cap \phi = \phi$

Commutative Laws:

i) $A \cup B = B \cup A$

ii) $A \cap B = B \cap A$

Associative Laws :

i) $(A \cup B) \cup C = A \cup (B \cup C)$

ii) $(A \cap B) \cap C = A \cap (B \cap C)$

- **Distributive Laws**
- **i)** A U (B∩C)= (AUB) ∩ (AUC)
- ii) A⋂(BUC) = (A⋂ B)U(A∩ C)

- **De-Morgan's Laws :**
- i) (AUB)'=A'∩ B'
- ii) (A∩B)' =A'UB'

Some of Important Results

- 1. **$A \cup B = \phi$ $A \Leftrightarrow \phi$ and $B = \phi$**
- **2. $A - B = \phi \Leftrightarrow$** $A \subseteq B$
- **3. $A - B =$** $A \cap B^c$.
- 4. $A \subseteq B \Leftrightarrow B^c \subseteq A^c$
- 5. $A - (B \cup C) = (A - B) \cap (A - C)$

  6. $A - (B \cap C) = (A - B) \cup (A - C)$

  7. $A \cap (B - C) = (A \cap B) - (A \cap C)$

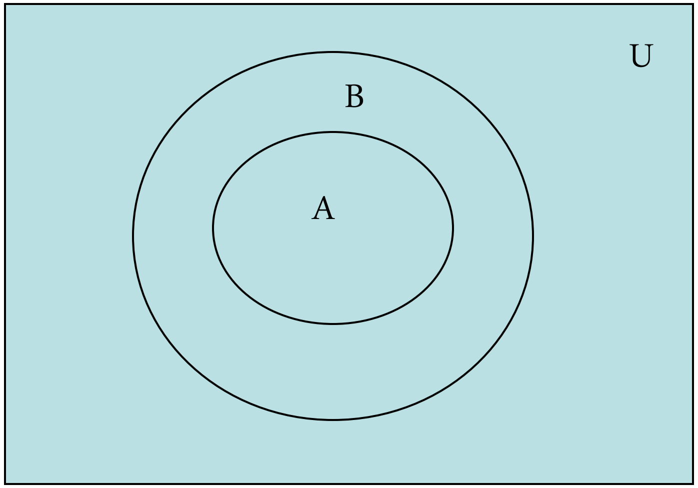
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) -$$

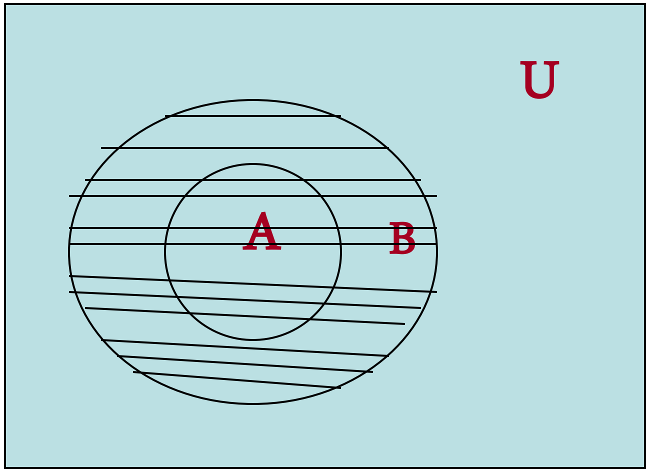
$$n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

Venn Diagrams

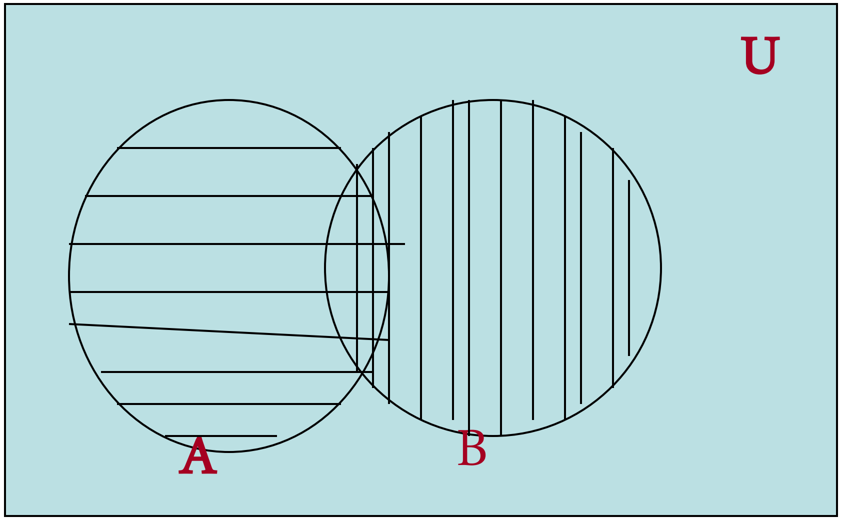
- A Sub sets $A \subset B$



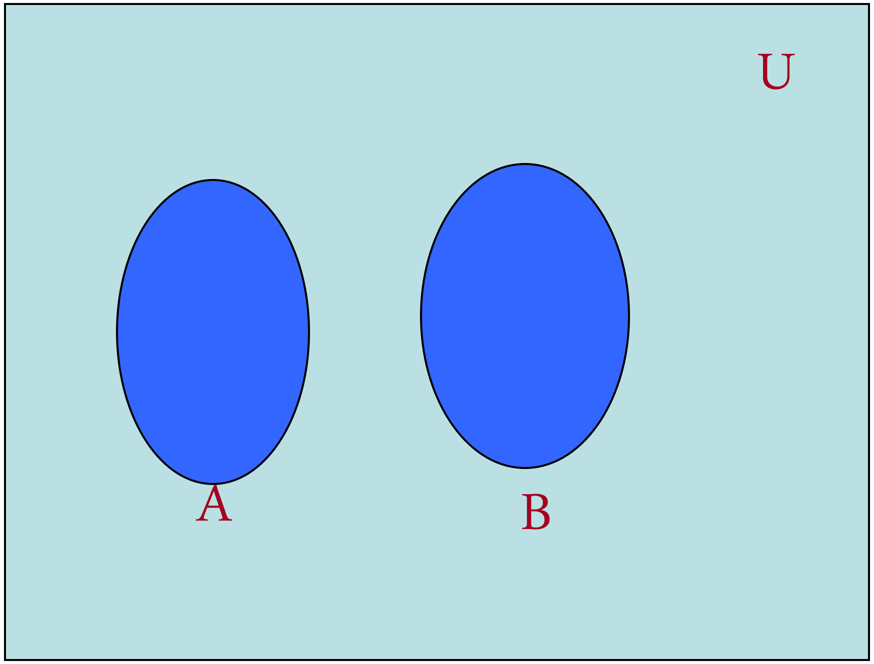
- **Union of Sets: Let AUB=B ,whole area represented by B represents AUB**



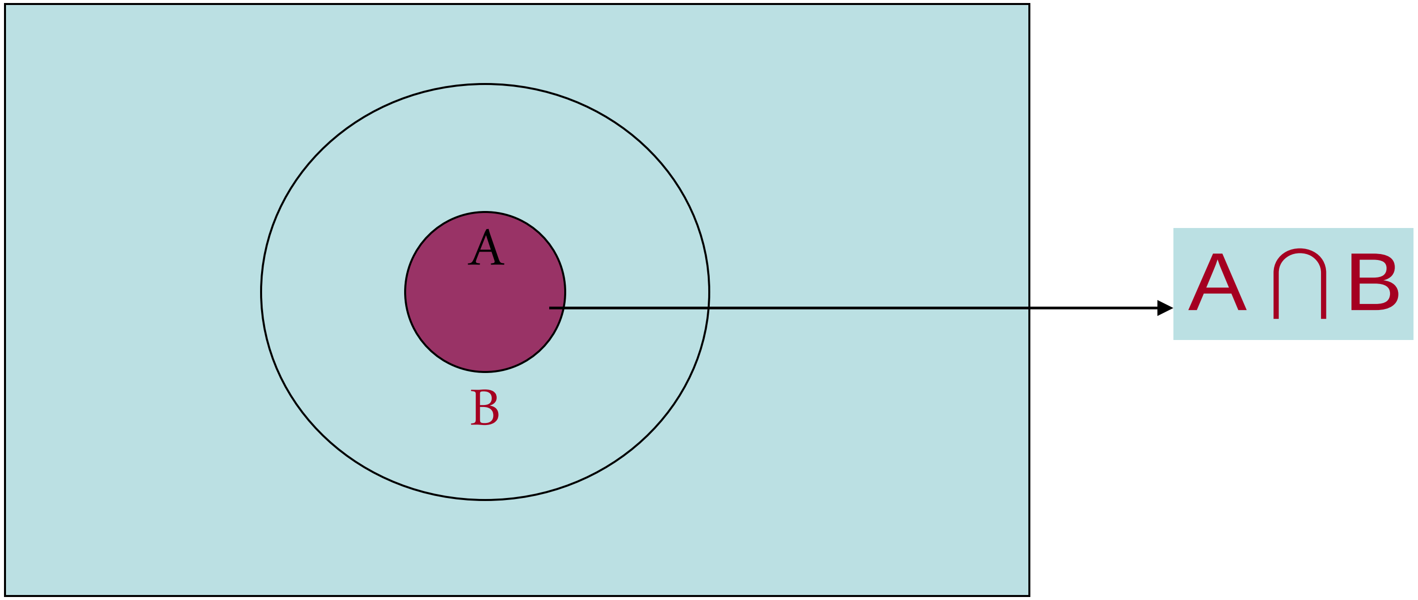
- **AU B when neither** $A \subset B$ nor $B \subset A$



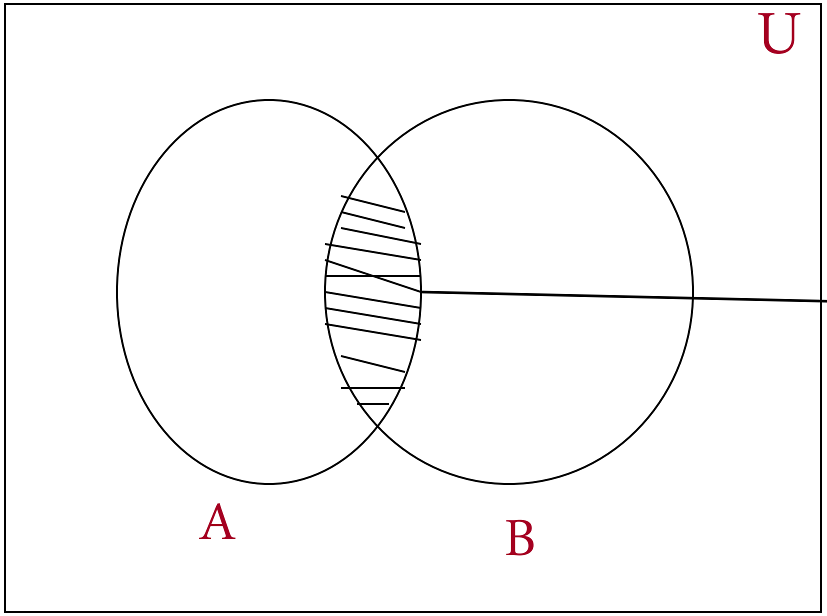
- **AUB when A and B are disjoint sets**



- **Intersection of Sets** $A \cap B$

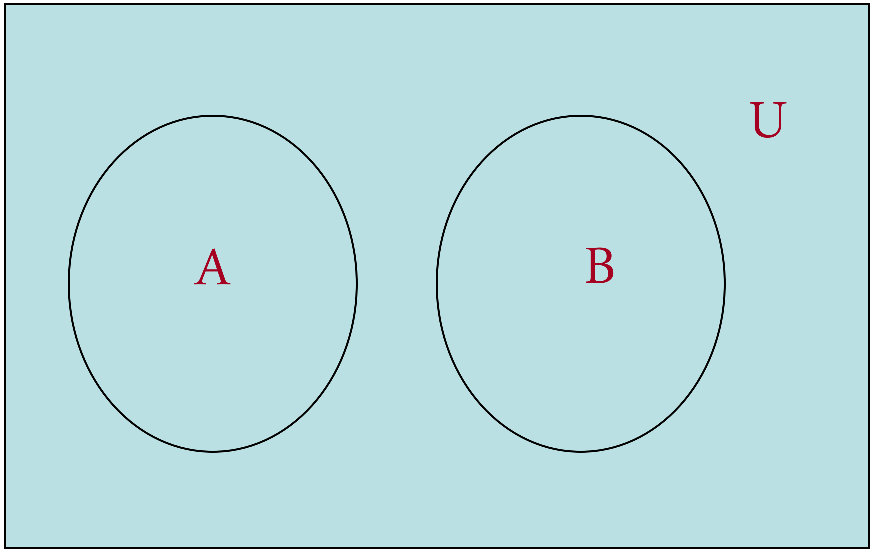


$A \cap B$

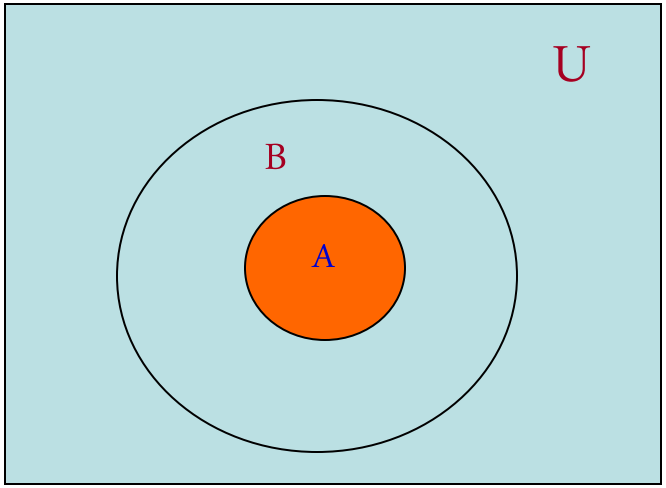


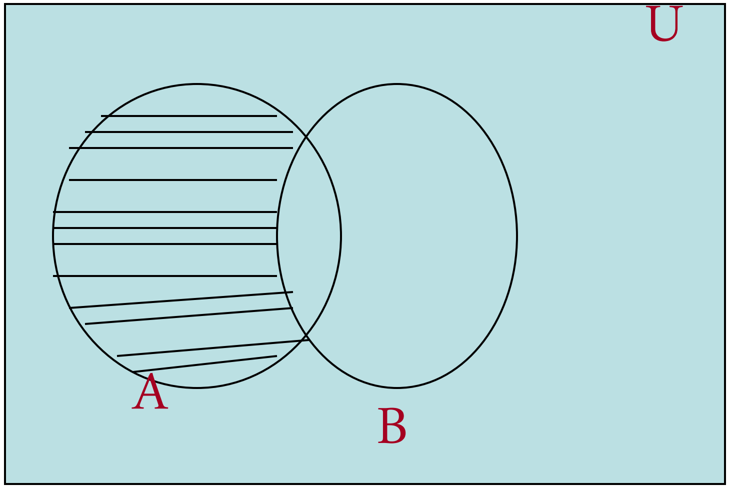
$A \cap B$

- **When A and B disjoints they are null set**

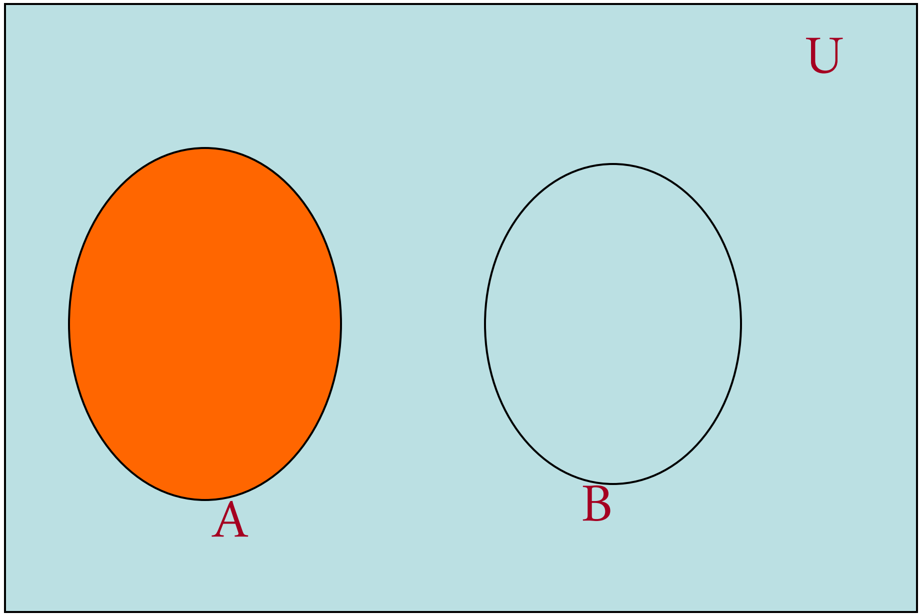


- **Difference of Sets**
- **A-B represents the are of A that is not in B**

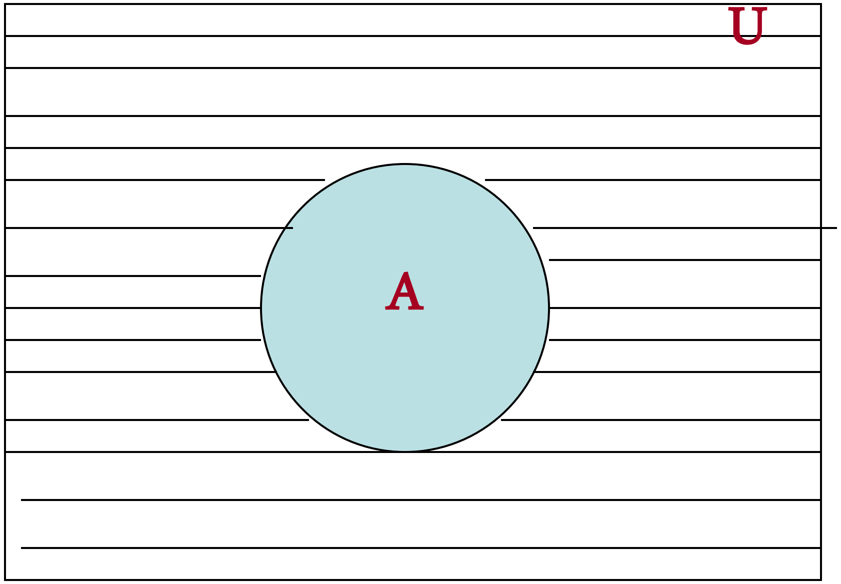




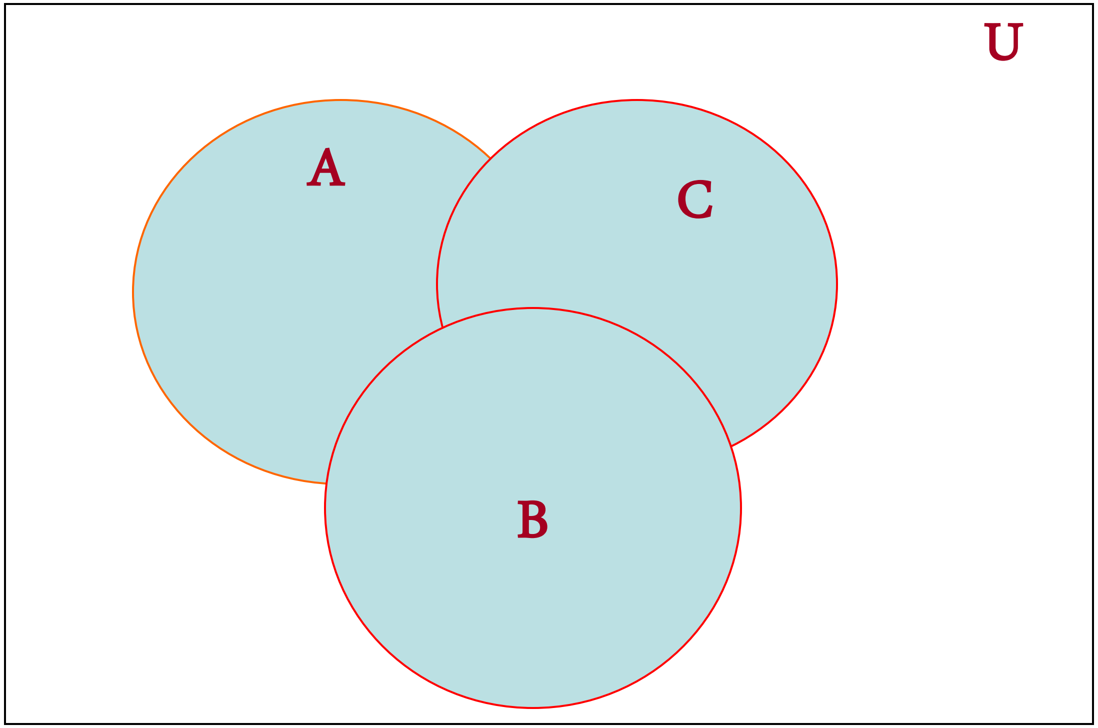
- **A-B ,When A and B are disjoint**



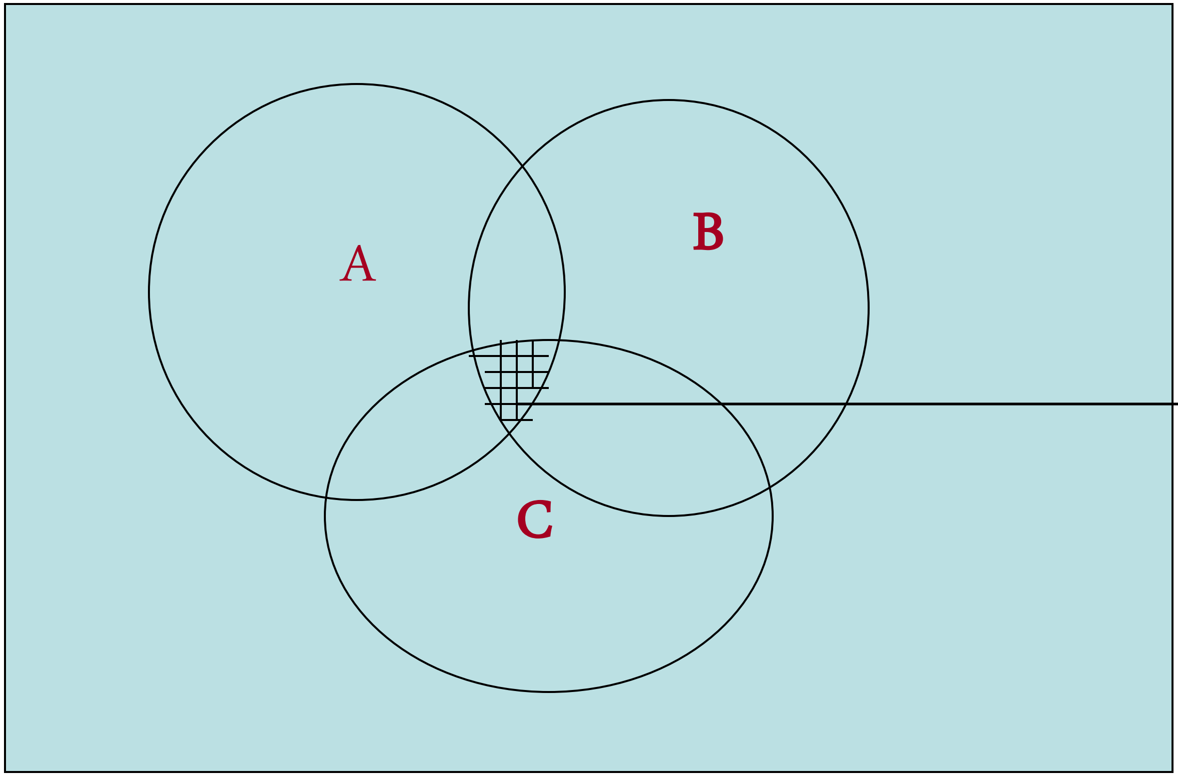
- **A' or A^c**



- **AU(BUC)**

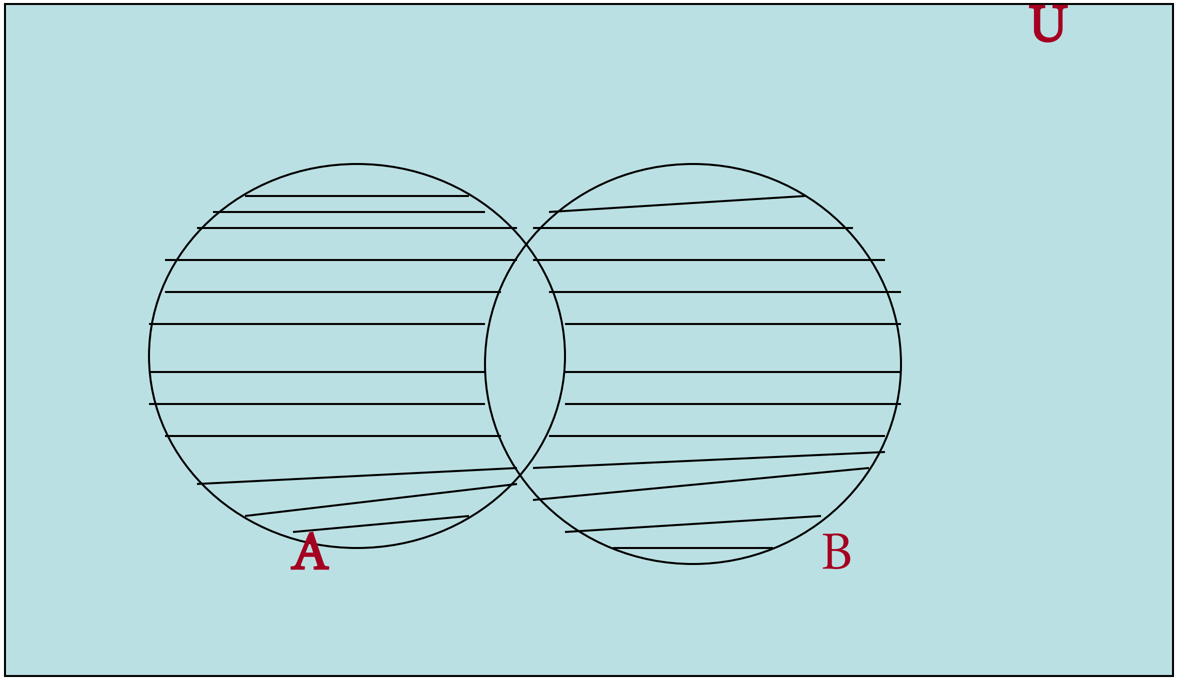


$(A \cap B) \cap C$



$(A \cap B) \cap C$

- **A∆B=(A-B)U(B-A)**



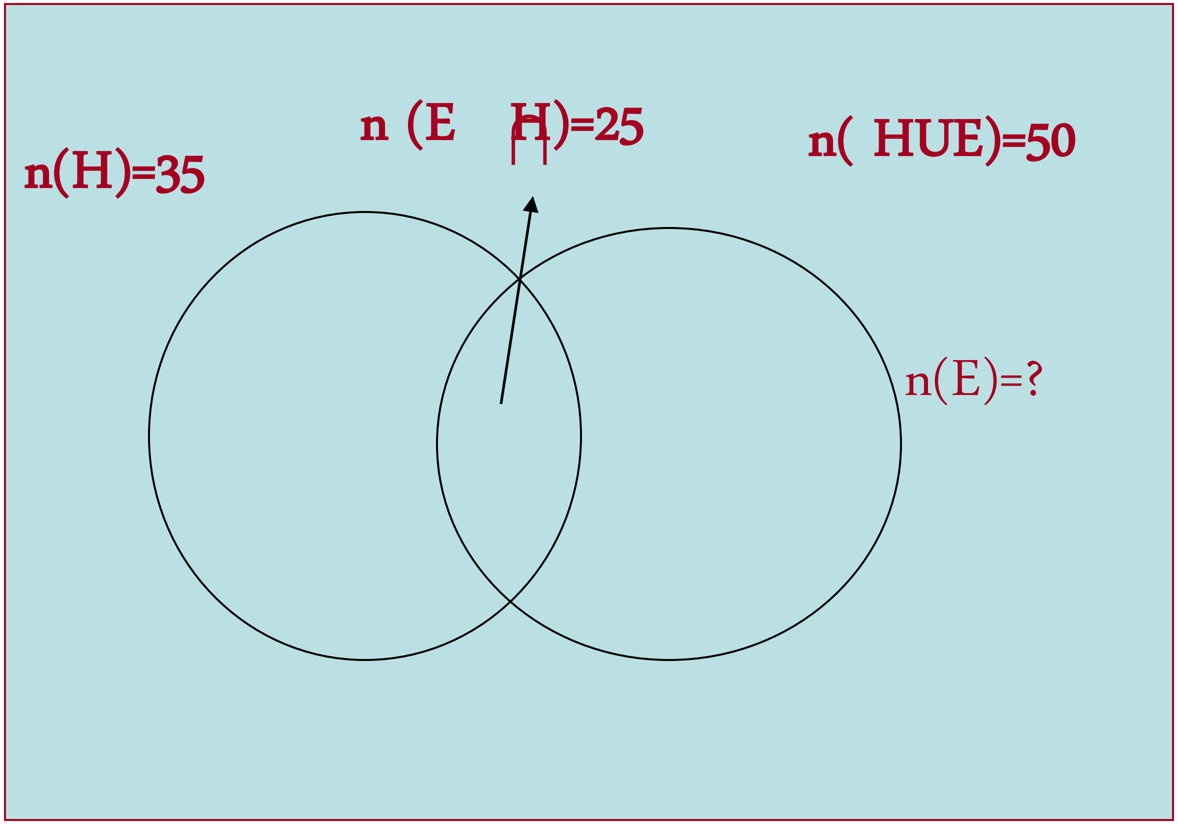
Example

- **In a school there are 20 teachers who teach mathematics or physics .of these ,12 teach mathematics and 4 teach physics and mathematics. How many teach Physics;**
- **Solution:**
- **$n(A \cup B) = n(A) + n(B) - n(A \cap B)$**

$= 20 - 12 + 4 = 12$

Example

- **In a group of 50 people ,35 speak Hindi ,25 speak both English as well as Hindi and all the speak at least one of two languages. How many speak English and not Hindi? How many people speak English?**



n(H)=35
n (E ∩ H)=25
n(HUE)=50
n(E)=?

Solution

- Given that n (HUE)=50,n(H)=35,
- n(H ∩ E)=25
- n (HUE)=n(H)+ n (E-H)
- 50=35+ n (E-H)=15
- Thus the number of persons speak English and not Hindi is 15
- Hence the no. of persons who speaks English=40
- n (HUE)=n (H) +n (E)-n (H ∩ E)
- 50=35+n(E)-25=40

Relations

- **Ordered Pair: Two elements a and b listed in a specific order pair, denoted by (a,b)**
- **Two order pairs (a,b) and (c,d) are said to be equal iff a=c and b=d**
- **Relation in a set : A relation between two sets A and B is a sub set of AXB and is denoted by R**
- **Thus** $R \subseteq A \times B$ **,we write x R y, iff(x,y) €R x R y read as x is related to y**

Domain and Range of Relation

- The Domain D of the relation R is defined as the set of all the first elements of the ordered pairs which belong to R i.e
- D={x: (x,y) €R for x €A}
- The range E of the relation R is defined as the set of all second elements of the ordered pairs which belong to R i.e
- E={y; (x,y) €R ,for y €B}

Different types of Relations

- **Inverse Relation: Let R be a Relation from the set A to the set B ,then the inverse relation $R^{-1}$from the set B to the set A is defined by**
- **R^{-1}={(b,a) : (a,b) €R}**

Example

- **Let A={1,2,3} ,B={a,b}**
- **And R={(1,a),(1,b),(3,a),(2,b)} be a relation from A to B**
- **Then inverse relation of R is**
- **R^{-1}={(a,1),(b,1),(a,3),(b,2)}**

- **Identity Relation:** Let R be a relation from the set A is said to be identity relation ,generally denoted by I_A, if
- I_A={(x,x) :x€A}
- Example: Let A={2,4,6} then
- I_A={(x,x):2,2)(4,4),(6,6)}is an identity relation from in A

- **Universal Relations :** A relation R from A to B is said to be universal relation if
- R=AXB
- Example : Let A={1,2,3} then
- R =AXA=
- ={(1,1)(1, 2),(1,3)(2,1)(2,2),(2,3)(3,1)
- (3,2),(3,3) } is a universal relation in A

- **Reflexive Relation :**Let R be a relation R in a set A, then R is called reflexive relation if (a,a) €R for all a€A
- In other words ,R is reflexive relation if every element of A is related itself.
- **Symmetric Relations:** Let R be a relation R in a set A, then R is called Symmetric relation if (a,b) €R $\Longleftrightarrow$ (b,a) €R

- **Transitive Relations:** **Let R be a relation R in a set A, then R is called transitive relation if (a,b) €R and (b,c) €R ⇄ (a,c) €R**
- **Equivalence Relations :Let R be a relation in set A .if**
- **1)R is reflexive**
- **2) R is Symmetric**
- **3.R is Transitive**

Definition

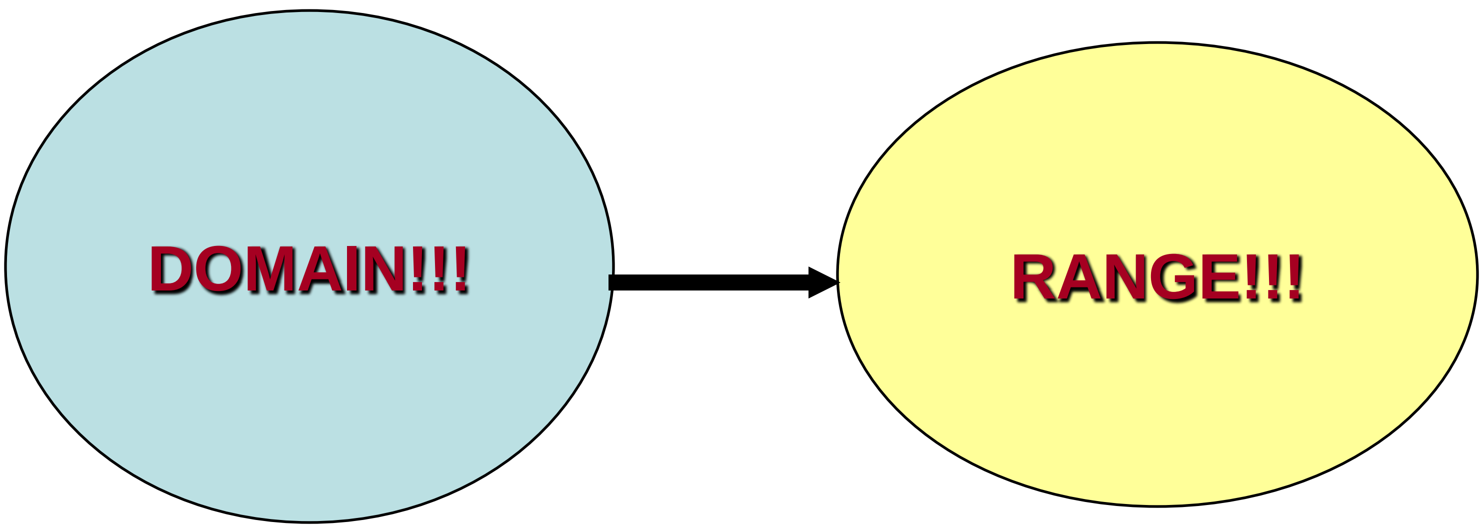
What is a function?

- The Set Definition of a Function:

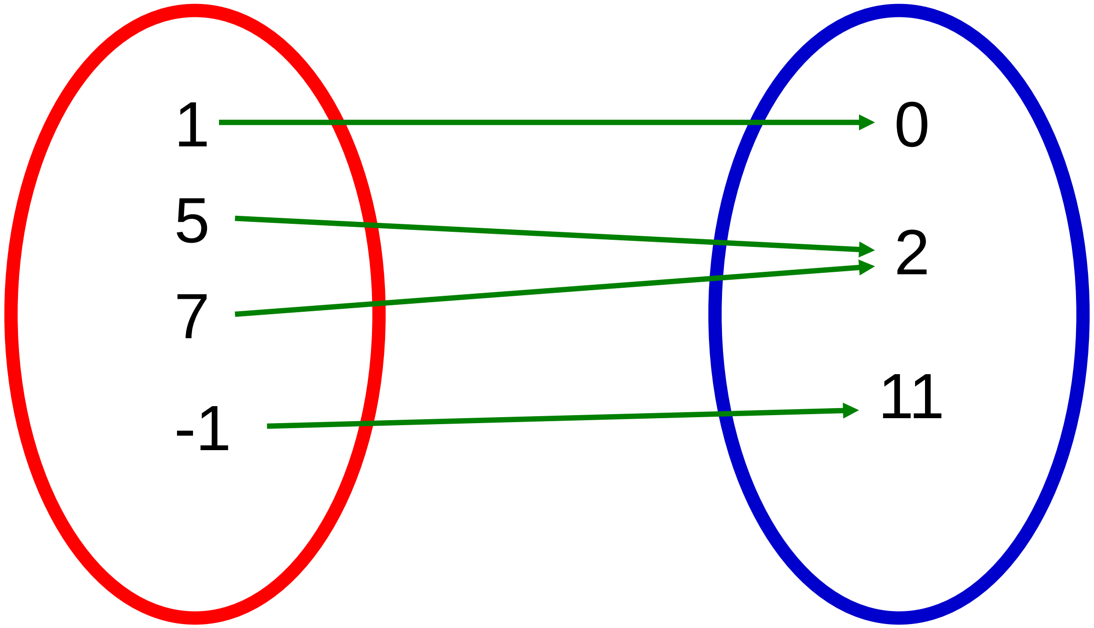
Given two non-empty sets X and Y, a function from X to Y is a correspondence or association that assigns each element of set X to exactly one element of set Y.

The Domain and the Range

- Domain is the STARTING POINT
- Range is the ENDING POINT



Example 1



Relation → {(1, 0), (5, 2), (7, 2), (-1, 11)}

Domain → {1, 5, 7, -1}

Range → {0, 2, 11}

Various types of Functions

- One-one function : A function f:A→B is said to be one-one if $f(a_1)=f(a_2)$,implies for all (a_1,a_2) **€A is also called injective.**
- **On to function (Surjective) :**

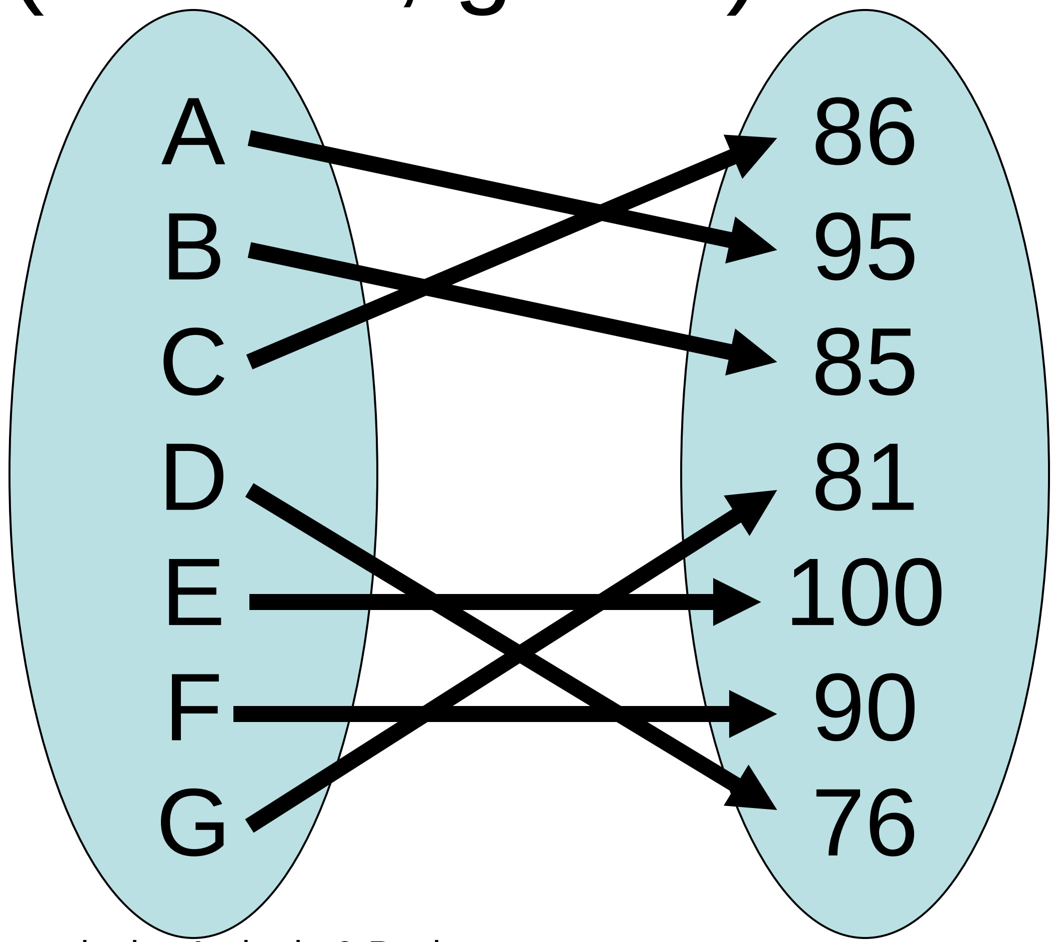
  Range of f =Co-domain
- **Bijective function : A function f; A to B is one-one and on-to function then it is called bijective function .**

- **Identity function:** Let A be a non-empty set .Then the function I is defined by I: A to A : I (x) = x for all x €A is called an identity function on A .It is a one-to-one onto function .

Constant function : Let f: A to B ,defined in such a way that all the elements in A have the same image in B ,then f is said to be constant function. The range of constant function is singleton.

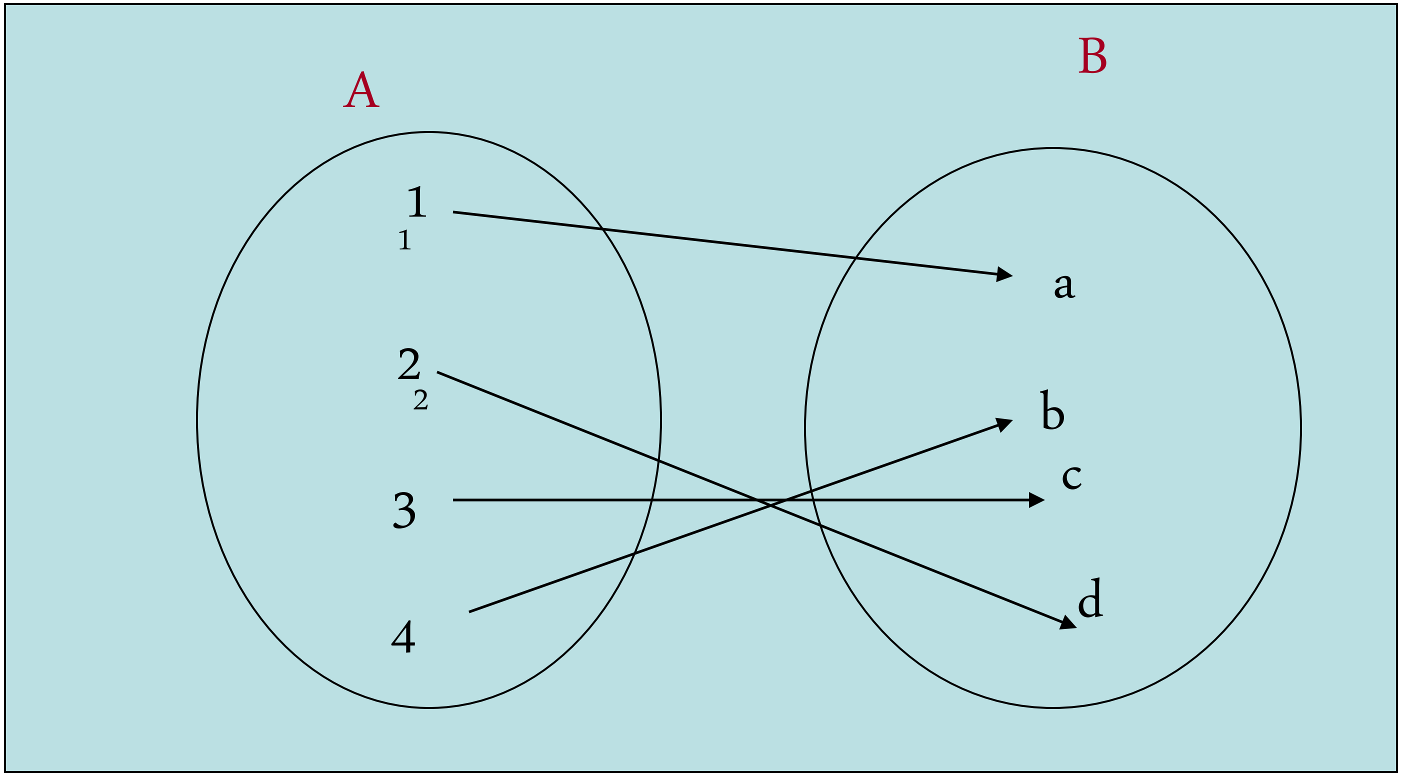
Example of CORRESPONDENCE (student, grade)

SET X	SET Y
A	95
B	85
C	86
D	76
E	100
F	90
G	81

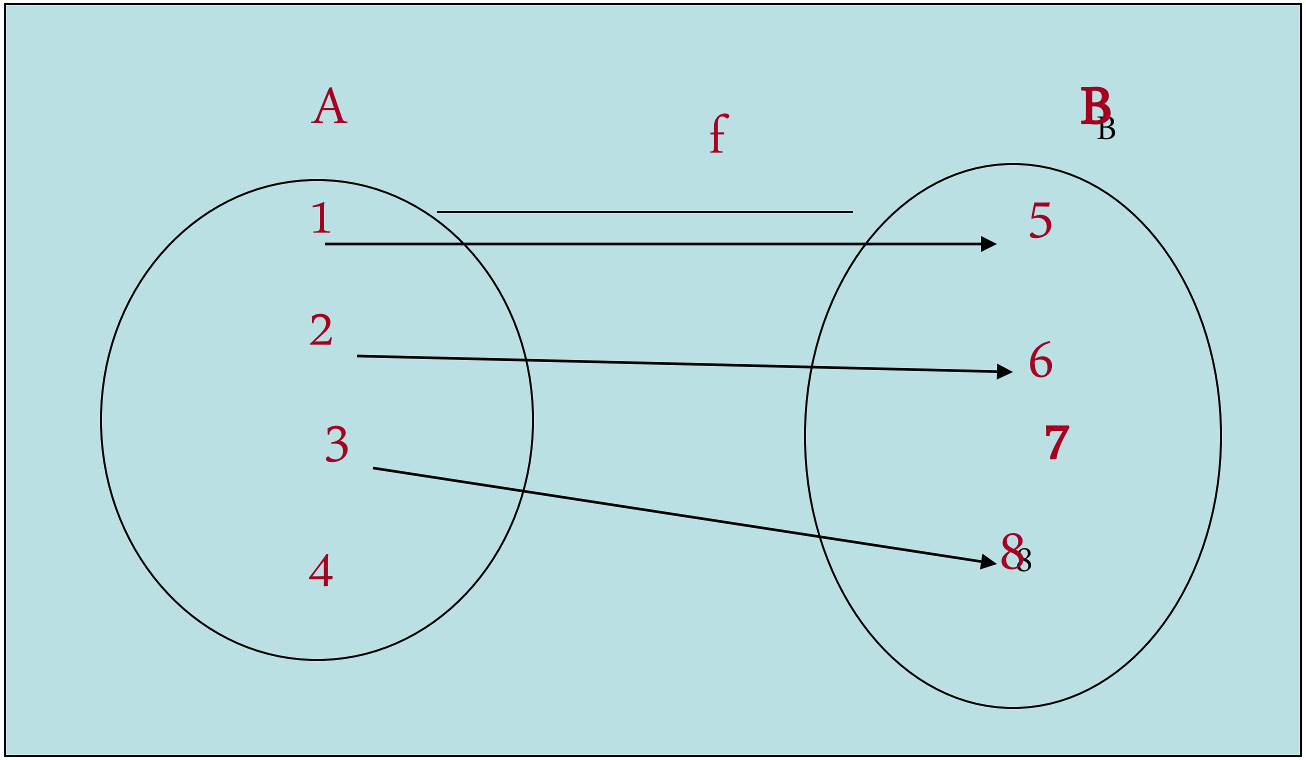


One-to-One CORRESPONDENCE

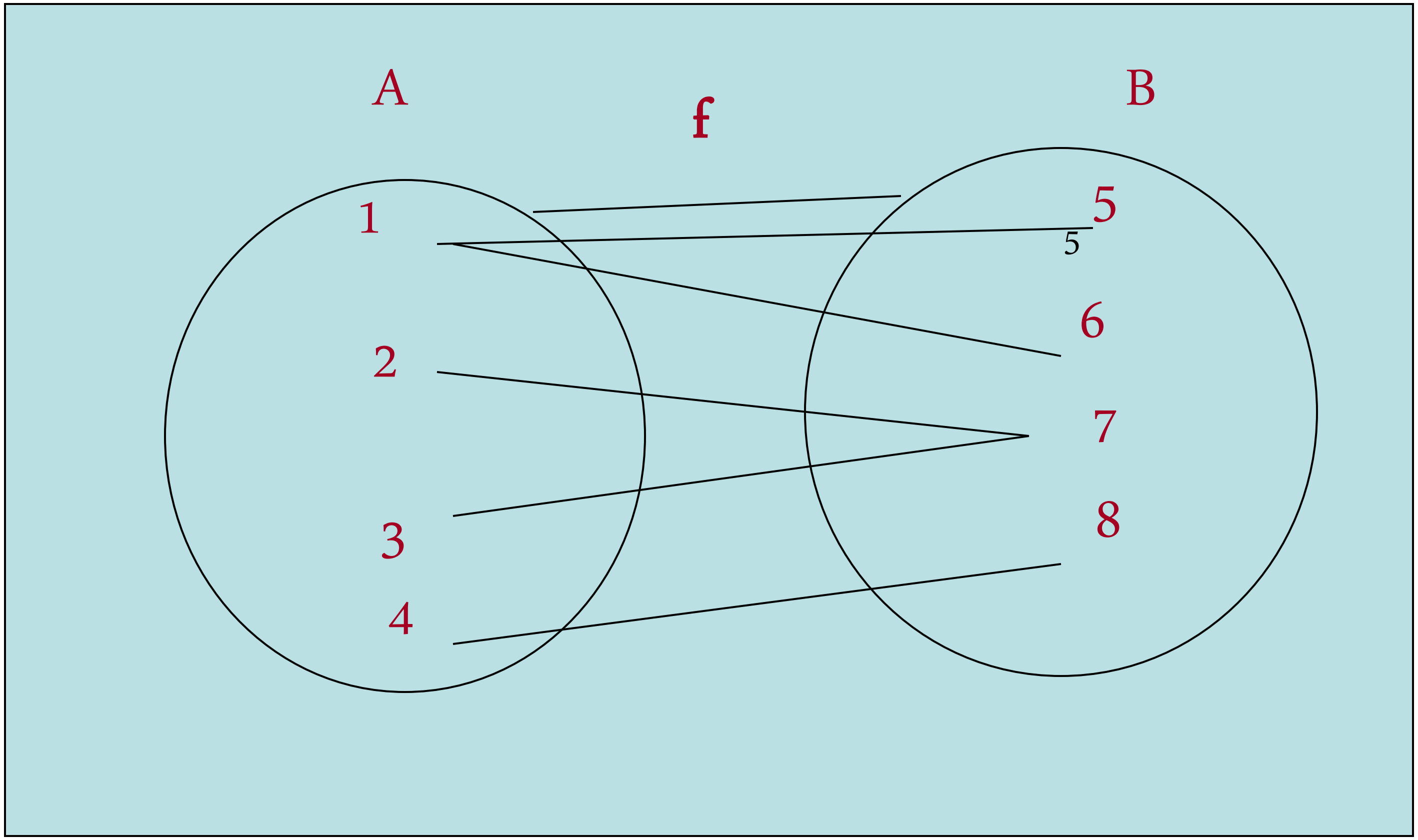
On to function



Examples of Not a Functions



Examples of Not a Functions



- **Equal functions**: Two functions f and g are said to be equal ,written as f=g ,if they have the same domain and they satisfy the condition f(x)=g(x)
- **Inverse function** :Let f be a one one onto function from A to B ,Let y be the arbitrary element of B ,we may define the function ,denoted by

f^{-1} :B to A : $f^{-1}(y)=x$ iff $f(x)=y$

- Composite functions: Let f; A to B and g: B to C be two functions from A to C which maps an element **x €A into g (f(x)) €C**

is called composite functions f and g and is written as g of

Example Let A ={1,3,5},B={3,9,15,21}

C={2,8,14,20,24}then

- $f=\{(1,3),(3,9),(5,15)\}$
- $g=\{(3,2),(9,8),(15,4),(21,20)\}$
- Then g o f is the function $=\{(1,2),(3,8),(5,14)\}$

Function Operations

$f(x) = x^2 + 2x + 1$ $g(x) = 3x + 2$

Domain?

$f(x) + g(x) = x^2 + 2x + 1 + 3x + 2$

$= \mathbf{x^2 + 5x + 3}$ $\{x : x \in R\}$

$f(x) - g(x) = (x^2 + 2x + 1) - (3x + 2)$

$= \mathbf{x^2 - x - 1}$ $\{x : x \in R\}$

$f(x) \bullet g(x) = (x^2 + 2x + 1) \bullet (3x + 2)$

$= \mathbf{3x^3}$

$= \mathbf{3x^3}$ $\{x : x \in R\}$

$$f(x) \div g(x) = \frac{(x^2 + 2x + 1)}{(3x + 2)}$$ $\{x : x \in R,\ x \neq \frac{-2}{3}\}$

Composite Functions

$f(x) = x^2 + 2x + 1$ $g(x) = 3x + 2$

Domain?

$f(g(x)) = f(3x+2)$

$= (3x+2)^2 + 2(3x+2) + 1$

$= 9x^2 + 12x + 4$

$= \mathbf{9x^2 + 18x + 9}$ $\{x : x \in R\}$

$g(f(x)) = g(x^2 + 2x + 1)$

$= 3(x^2 + 2x + 1) + 2$

$= 3x^2 + 6x + 3$

$= \mathbf{3x^2 + 6x + 5}$ $\{x : x \in R\}$

Composite Functions

$f(x) = x^2 - 4x + 5$ $g(x) = x - 3$

Domain?

$$f(g(x)) = f(x-3)$$
$$= (x-3)^2 - 4(x-3) + 5$$
$$= x^2 - 6x + 9$$
$$= \mathbf{x^2 - 10x + 26}$$

$$\{x : x \in R\}$$

$$g(f(x)) = g(x^2 - 4x + 5)$$
$$= (x^2 - 4x + 5) - 3$$
$$= x^2 - 4x + 5$$
$$= \mathbf{x^2 - 4x + 2}$$

$$\{x : x \in R\}$$

Composite Functions

$f(x) = x^2 + 3x + 5$ $\quad g(x) = \sqrt{x-3}$

Domain?

$$f(g(x)) = f(\sqrt{x-3})$$

$$= (\sqrt{x-3})^2 + 3(\sqrt{x-3}) + 5$$

$$= x-3 + 3\sqrt{x-3} + 5$$

$$= x+2 + 3\sqrt{x-3}$$

$x - 3 \geq 0$

$x \geq 3$

$$[3, +\infty)$$

Composite Functions

$f(x) = x^2 + 3x + 5$ $\quad g(x) = \sqrt{x-3}$

Domain?

$$g(f(x)) = g(x^2 + 3x + 5)$$

$$= \sqrt{(x^2 + 3x + 5) - 3}$$

$$= \sqrt{x^2 + 3x + 5 - 3}$$

$$= \sqrt{x^2 + 3x + 2}$$

$x^2 + 3x + 2 \geq 0$

$(x + 2)(x + 3) \geq 0$

$$(-\infty, -3] \cup [-2, +\infty)$$

- 1.If P = {1,2,3,4} and Q = {2,4,6} then PUQ
- A) {1,2,3,6}
- B) {1,4,6}
- C) **{1,2,3,4,6}**
- D) None of these.

- 1.If P = {1,2,3,4} and Q = {2,4,6} then PUQ
- A) {1,2,3,6}
- B) {1,4,6}
- C) **{1,2,3,4,6}**
- D) None of these.

- 2.If A has 70 elements, B has 32 elements and $A \cap B$ has 22 elements then AUB is
- A) 60
- B) 124
- C) **80**.
- D) None of these

- 2.If A has 70 elements, B has 32 elements and A ∩ B has 22 elements then AUB is
- A) 60
- B) 124
- C) **80**.
- D) None of these

- 3. The number of subsets of the set {1,2,3,4} is
- A) 13
- B) 12
- C) **16**
- D) 15

- 3. The number of subsets of the set {1,2,3,4} is
- A) 13
- B) 12
- C) **16**
- D) 15

- 4. If A = {1, 2, 3, 4} and B = {5, 6, 7}, then cardinal number of A X B is
- A) 4
- B) 7
- C) **12**
- D) None of these

- 4. If A = {1, 2, 3, 4} and B = {5, 6, 7}, then cardinal number of A X B is
- A) 4
- B) 7
- C) **12**
- D) None of these

- 5. In a group of 20 children, 8 drink tea but not coffee and 13 like tea. The number of children drinking coffee but not tea is
- A) 6
- B) **7**
- C) 1
- D) None of these

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- A) 6
- B) **7**
- C) 1
- D) None of these

- 6. Find the *f o g* for the functions f (x) =x^2, g (x) = x+ 1
- A) x^2 (x+1)
- B) x^2
- C) x+1
- D) **$(x+1)^2$**

- 6. Find the *f o g* for the functions f (x) =x^2, g (x) = x+ 1
- A) x^2 (x+1)
- B) x^2
- C) x+1
- D) **$(x+1)^2$**

- 7. If P is a set of natural number then
- $P \cap P'$ is
- A) ϕ.

  B) 0
- C) Sample Space
- D) $(P \cup P')'$

- 7. If P is a set of natural number then
- $P \cap P'$ is
- A) ϕ.

  B) 0
- C) Sample Space
- D) (PUP')'

- 8. "Is greater than" over the set of all natural number if known as
- A) **Transitive**
- B) Symmetric
- C) Reflexive
- D) Equivalence

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- D) Equivalence

- 9. A function f(x) is an even function, if
- A) $-f(x) = f(x)$**)**
- B) **$f(-x) = f(x)$**
- C) $f(-x) = -f(x)$
- D) None of these

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- A) $-f(x) = f(x)$)
- B) **$f(-x) = f(x)$**
- C) $f(-x) = -f(x)$
- D) None of these

- 10.If A = {1, 2, 3, 4} and B = {5, 6, 7}, then cardinal number of the set A×B is ______
- A) 7
- B) 1
- C) **12**
- D) None of these

- 10.If A = {1, 2, 3, 4} and B = {5, 6, 7}, then cardinal number of the set A×B is _____
- A) 7
- B) 1
- C) **12**
- D) None of these

THE END

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