



Sequences and Series





Arithmetic Sequences

Definitions Sequence: a list of numbers in a specific order.

• Term: each number in a sequence

Definitions Arithmetic Sequence: a sequence in which each term after the first term is found by adding a constant, called the common difference (d), to the previous term.

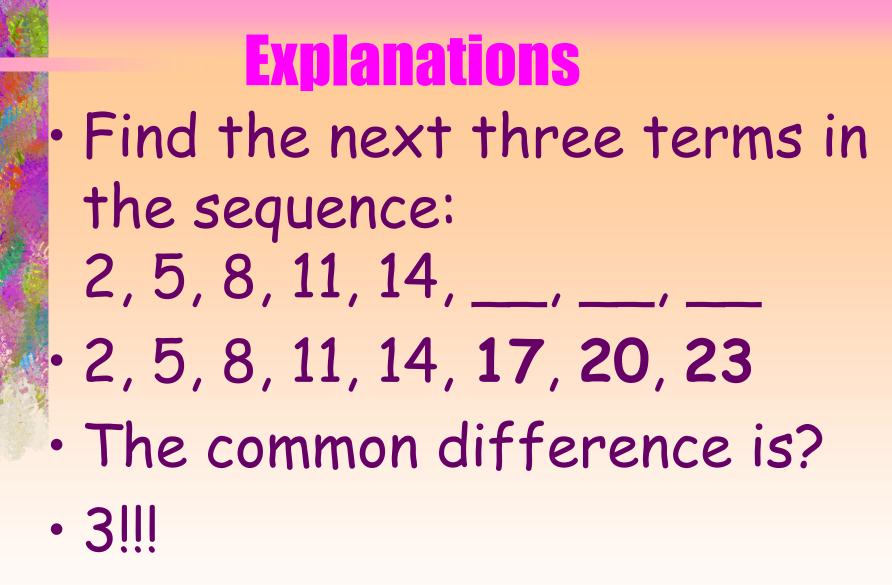
Explanations · 150, 300, 450, 600, 750... The first term of our sequence is 150, we denote the first term as a_1 .

- What is a₂?
- a₂: 300 (a₂ represents the 2nd term in our sequence)

Explanations• $a_3 = ?$ $a_4 = ?$ $a_5 = ?$ • $a_3 : 450$ $a_4 : 600$ $a_5 : 750$

 a_n represents a general term (nth term) where n can be any number.

Explanations Sequences can continue forever. We can calculate as many terms as we want as long as we know the common difference in the sequence.



Explanations To find the common difference (d), just subtract any term from the term that follows it.

• FYI: Common differences can be negative.

 What if I wanted to find the 50th (a_{50}) term of the sequence 2, 5, 8, 11, 14, ...? Do I really want to add 3 continually until I get there? • There is a formula for finding the nth term.

 Let's see if we can figure the formula out on our own.

• $a_1 = 2$, to get a_2 I just add 3 once. To get a_3 I add 3 to a_1 twice. To get a_4 I add 3 to a_1 three times.

- What is the relationship between the term we are finding and the number of times I have to add d?
- The number of times I had to add is one less then the term I am looking for.

Formula • So if I wanted to find a₅₀ then how many times would I have to add 3?

- 49
- If I wanted to find a₁₉₃ how many times would I add 3?
- · 192

 So to find a₅₀ I need to take d, which is 3, and add it to my a_1 , which is 2, 49 times. That's a lot of adding. But if we think back to elementary school, repetitive adding is just multiplication.

Formula • 3 + 3 + 3 + 3 + 3 = 15 We added five terms of three, that is the same as multiplying 5 and 3. So to add three forty-nine times we just multiply 3 and 49

 So back to our formula, to find a_{50} we start with 2 (a_1) and add 3.49. (3 is d and 49 is one less than the term we are looking for) So... $\cdot a_{50} = 2 + 3(49) = 149$

Formula • a₅₀ = 2 + 3(49) using this formula we can create a general formula.

 a₅₀ will become a_n so we can use it for any term.

• 2 is our a_1 and 3 is our d.

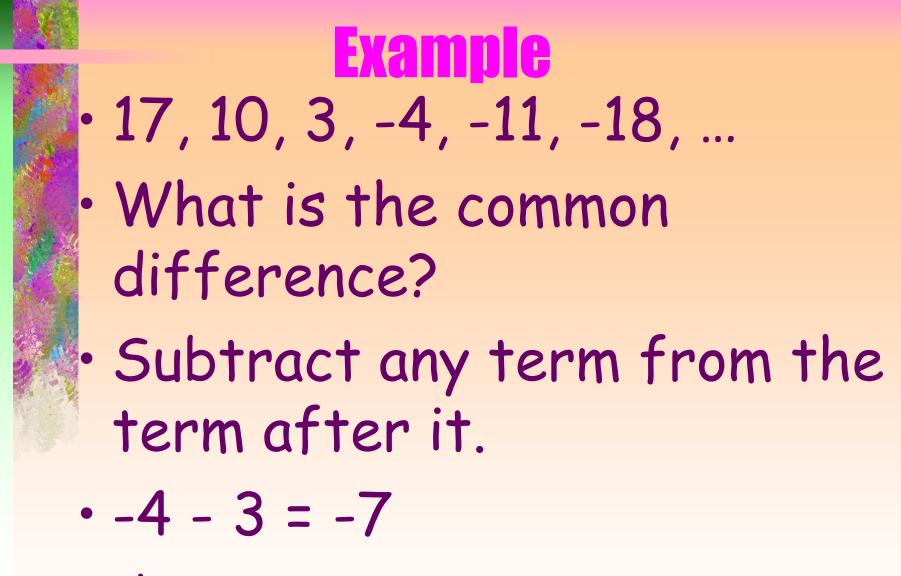
Formula $a_{50} = 2 + 3(49)$ • 49 is one less than the term we are looking for. So if I am using n as the term I am looking for, I multiply d by n - 1.

 Thus my formula for finding any term in an arithmetic sequence is $a_n = a_1 + d(n-1)$. All you need to know to find any term is the first term in the sequence (a_1) and the common difference.

• Let's go back to our first example . Find therm number 16?

• $a_n = a_1 + d(n-1)$

- We want to find a_{16} . What is
 - a_1 ? What is d? What is n-1?
- a₁ = **150**, d = **150**, n -1 = 16 - 1 = **15**
- So a₁₆ = 150 + 150(15) =
- •\$2400



• d = - 7



Definition • 17, 10, 3, -4, -11, -18, ...

 Arithmetic Means: the terms between any two nonconsecutive terms of an arithmetic sequence.

Arithmetic Means • 17, 10, 3, -4, -11, -18, ... Between 10 and -18 there are three arithmetic means 3, -4, -11. Find three arithmetic means

between 8 and 14.

Arithmetic Means
So our sequence must look like 8, _____, ____, 14.
In order to find the means

we need to know the common difference. We can use our formula to find it.

Arithmetic Means • 8, ____, 14 $\cdot a_1 = 8, a_5 = 14, \& n = 5$ $\cdot 14 = 8 + d(5 - 1)$ $\cdot 14 = 8 + d(4)$ subtract 8 • 6 = 4d divide by 4 $\cdot 1.5 = d$

Arithmetic Means • 8, ____, ___, 14 so to find our means we just add 1.5 starting with 8. • 8, 9.5, 11, 12.5, 14

Additional Example • 72 is the <u>term of the</u> sequence -5, 2, 9, ...

 We need to find 'n' which is the term number.

• 72 is a_n , -5 is a_1 , and 7 is d. Plug it in.

Additional Example \cdot 72 = -5 + 7(n - 1) • 72 = -5 + 7n - 7 \cdot 72 = -12 + 7n • 84 = 7n • n = 12 72 is the 12th term.





Arithmetic Series The African-American celebration of Kwanzaa involves the lighting of candles every night for seven nights. The first night one candle is lit and blown out.

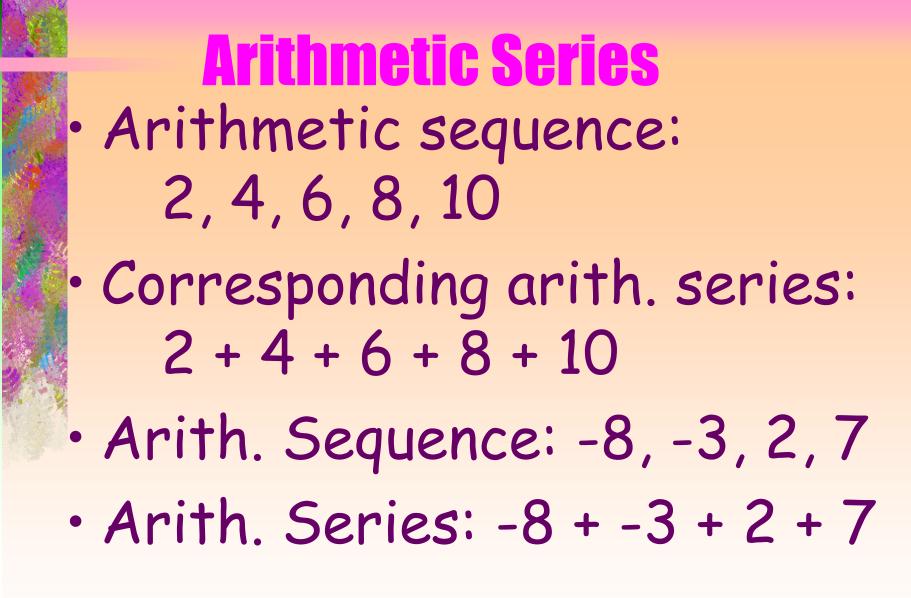
Arithmetic Series The second night a new candle and the candle from the first night are lit and blown out. The third night a new candle and the two candles from the second night are lit and blown out.

Arithmetic Series This process continues for the seven nights. We want to know the total number of lightings during

the seven nights of celebration.

Arithmetic Series The first night one candle was lit, the 2nd night two candles were lit, the 3rd night 3 candles were lit, etc. So to find the total number of lightings we would add: 1 + 2 + 3 + 4 + 5 + 6 + 7

Arithmetic Series •1+2+3+4+5+6+7=28 • Series: the sum of the terms in a sequence. Arithmetic Series: the sum of the terms in an arithmetic sequence.



 Arithmetic Series
 S_n is the symbol used to represent the first 'n' terms of a series.

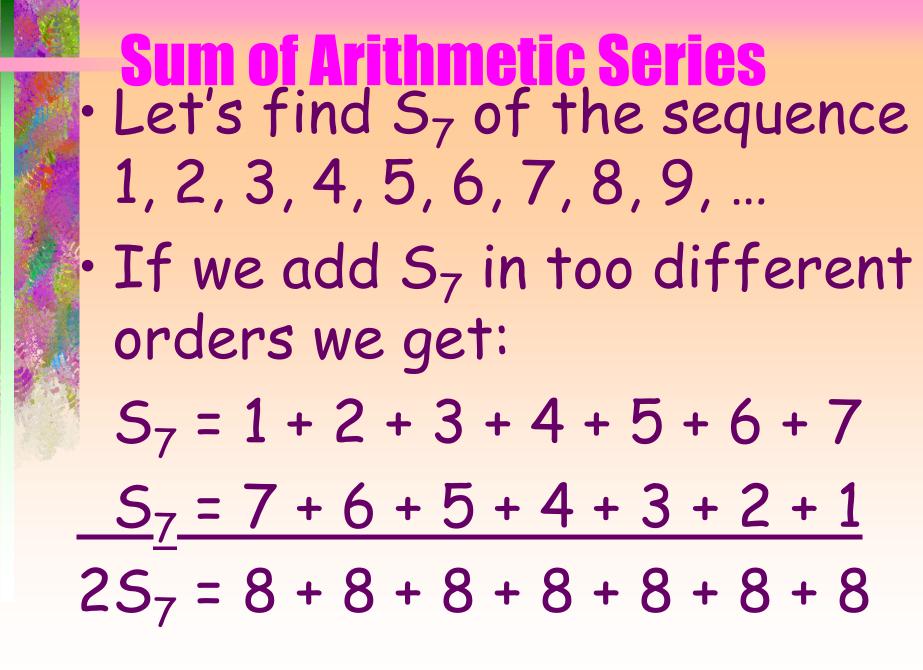
• Given the sequence 1, 11, 21, 31, 41, 51, 61, 71, ... find S₄

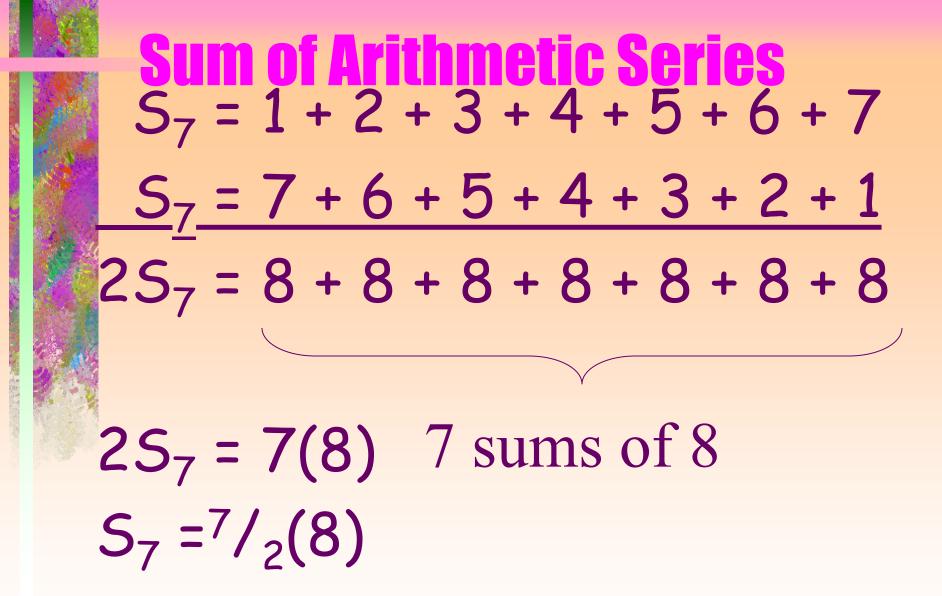
We add the first four terms
 1 + 11 + 21 + 31 = 64

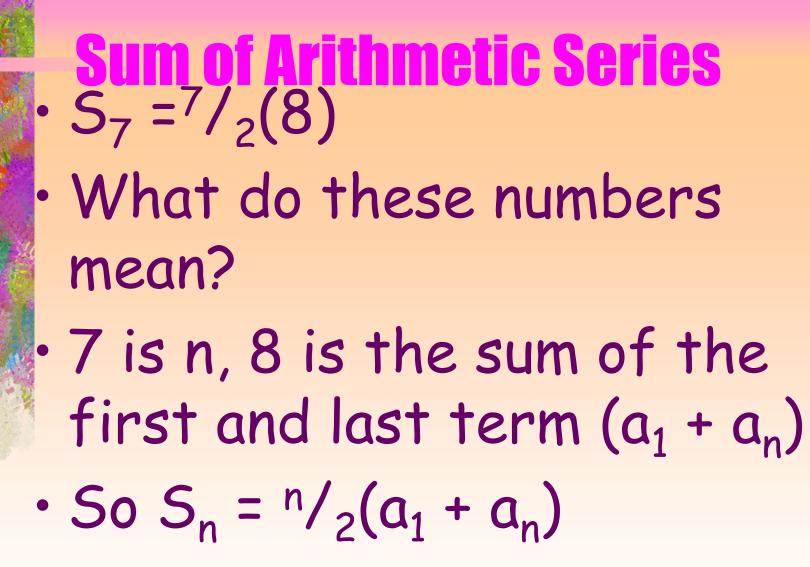
Arithmetic Series • Find S_8 of the arithmetic sequence 1, 2, 3, 4, 5, 6, 7, 8, 9,10,... •1+2+3+4+5+6+7+8= • 36

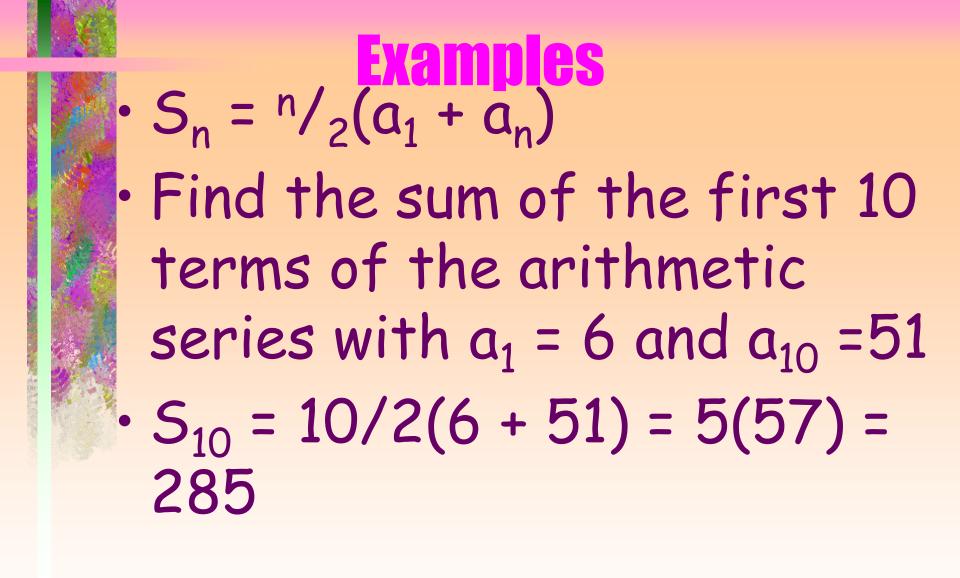
Arithmetic Series What if we wanted to find S_{100} for the sequence in the last example. It would be a pain to have to list all the terms and try to add them up.

Let's figure out a formula!! :)









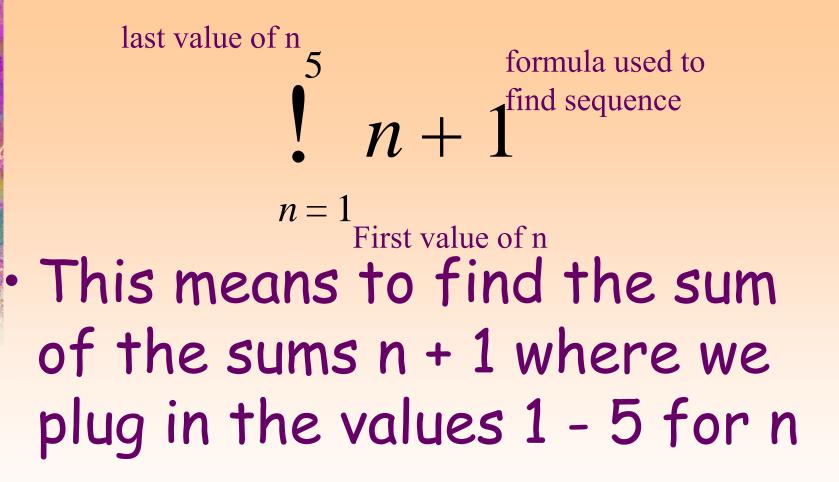
• Find the sum of the first 50 terms of an arithmetic series with $a_1 = 28$ and d = -4• We need to know n, a_1 , and **a**₅₀. • n= 50, $a_1 = 28$, $a_{50} = ??$ We

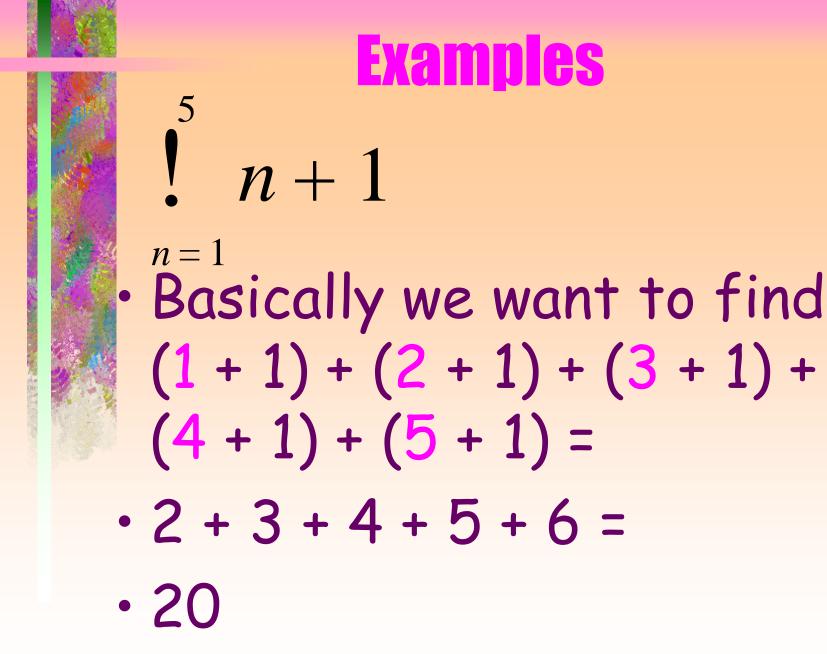
have to find it.

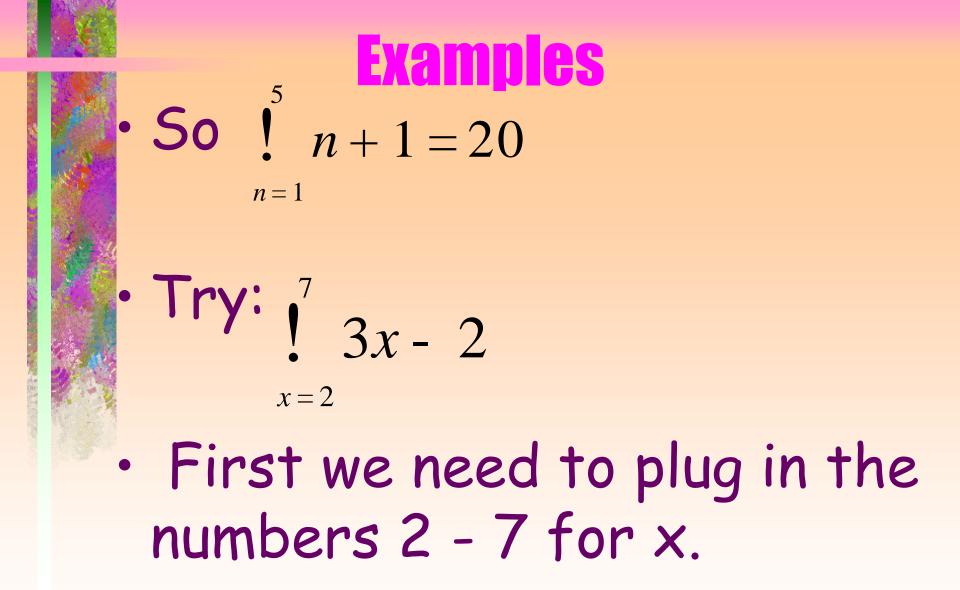
Examples $a_{50} = 28 + -4(50 - 1) =$ 28 + -4(49) = 28 + -196 =-168 • So n = 50, a₁ = 28, & a_n =-168 • S₅₀ = (50/2)(28 + -168) = 25(-140) = -3500

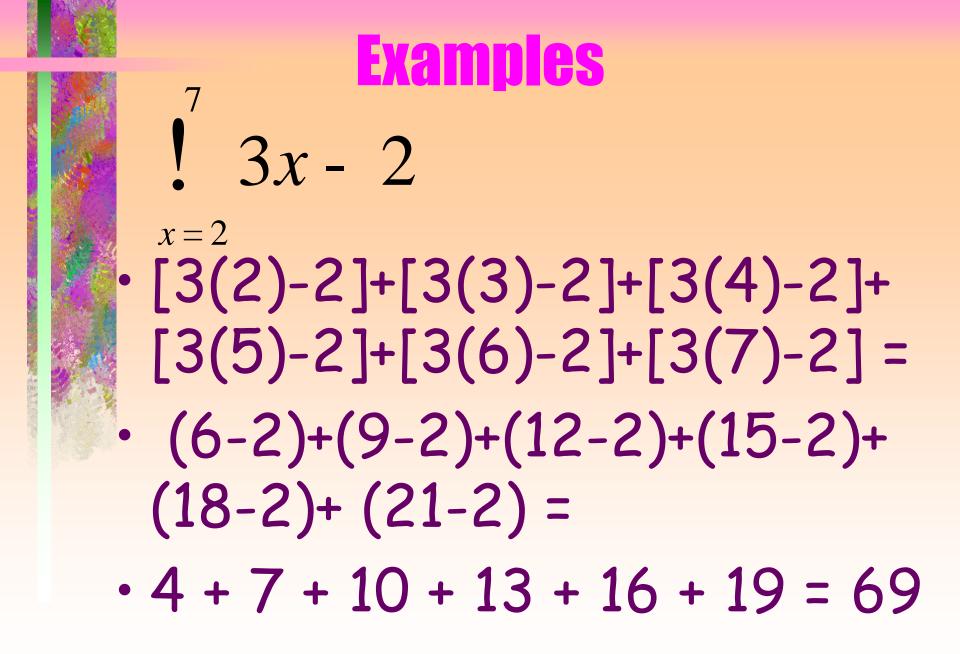
Examples To write out a series and compute a sum can sometimes be very tedious. Mathematicians often use the greek letter sigma & summation notation to simplify this task.

Examples













Geometric Sequences

GeometricSequence What if your pay check started at \$100 a week and doubled every week. What would your salary be after four weeks?

• Starting \$100. After one week - \$200 After two weeks - \$400 After three weeks - \$800 After four weeks - \$1600. These values form a geometric sequence.

Geometric Sequence Geometric Sequence: a sequence in which each term after the first is found by multiplying the previous term by a constant value called the common ratio.

Geometric Sequence Find the first five terms of the geometric sequence with $a_1 = -3$ and common ratio (r) of 5. ·-3, -15, -75, -375, -1875

• Find the common ratio of the sequence 2, -4, 8, -16, 32, ...

 To find the common ratio, divide any term by the previous term.

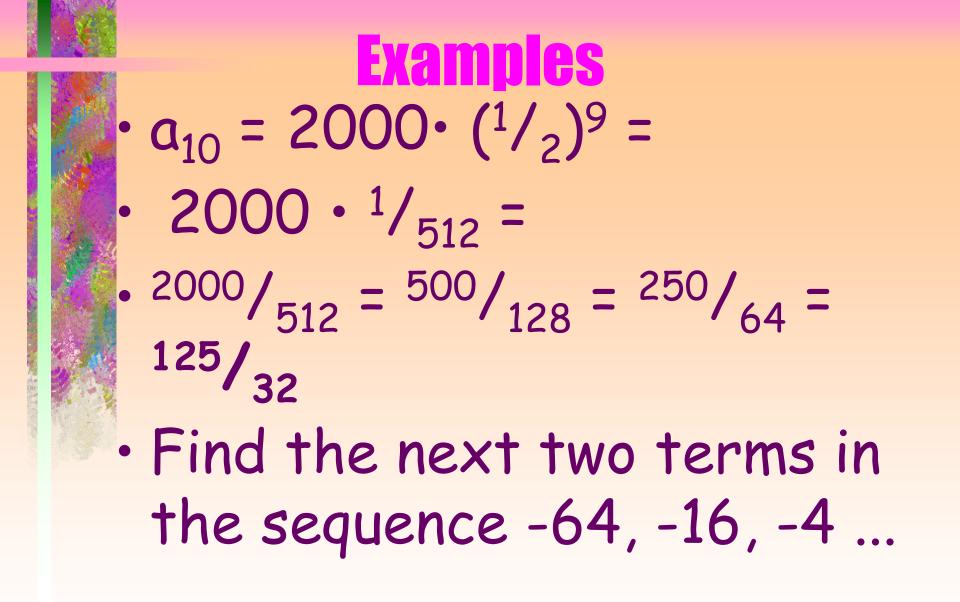
Geometric Sequence Just like arithmetic sequences, there is a formula for finding any given term in a geometric sequence. Let's figure it out using the pay check example.

• To find the 5th term we look 100 and multiplied it by two four times.

 Repeated multiplication is represented using exponents.

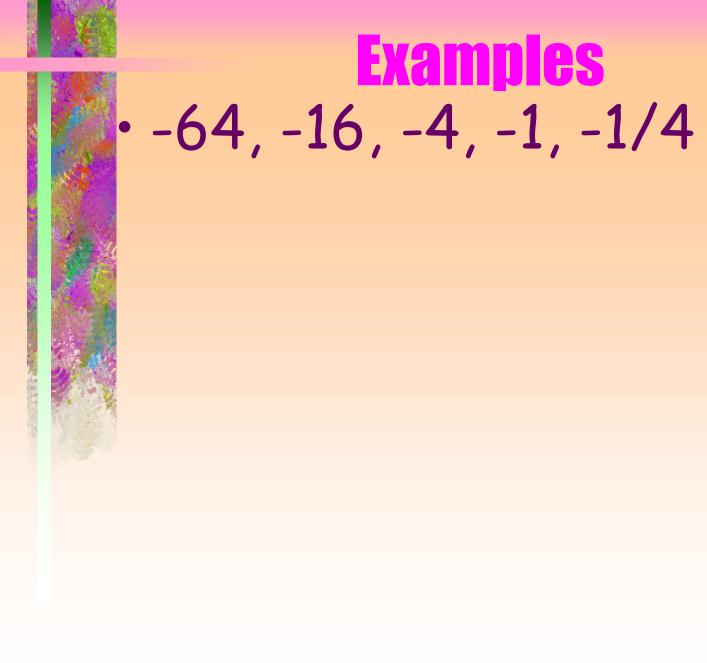
Geometric Sequence Basically we will take \$100 and multiply it by 2⁴ $\cdot a_5 = 100 \cdot 2^4 = 1600$ • A_5 is the term we are looking for, 100 was our a_1 , 2 is our common ratio, and 4 is n-1.

Examples Thus our formula for finding any term of a geometric sequence is $a_n = a_1 \cdot r^{n-1}$ Find the 10th term of the geometric sequence with $a_1 =$ 2000 and a common ratio of 1/2.



Examples • -64, -16, -4, ____, We need to find the common ratio so we divide any term by the previous term. $\cdot -16/-64 = 1/4$

 So we multiply by 1/4 to find the next two terms.



Geometric Means Just like with arithmetic sequences, the missing terms between two nonconsecutive terms in a geometric sequence are called geometric means.

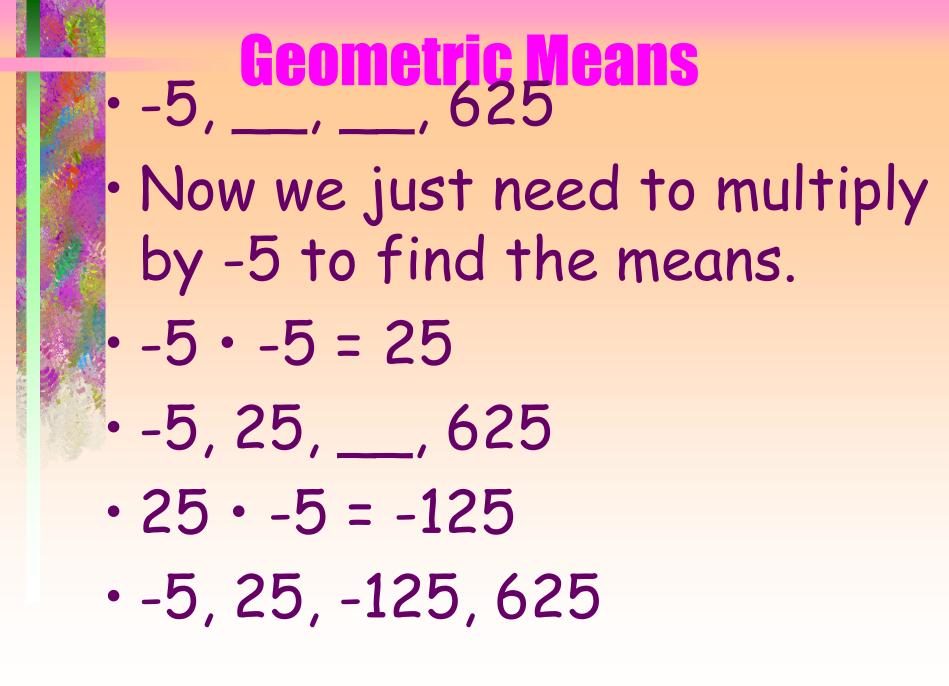
Geometric Means Looking at the geometric sequence 3, 12, 48, 192, 768 the geometric means between 3 and 768 are 12, 48, and 192.

 Find two geometric means between -5 and 625.

Geometric Means • -5, ____, 625 We need to know the common ratio. Since we only know nonconsecutive terms we will have to use the formula and work backwards.

Geometric Means • -5, ____, 625 • 625 is a₄, -5 is a₁. $\cdot 625 = -5 \cdot r^{4-1}$ divide by -5 • $-125 = r^3$ take the cube root of both sides

• -5 = r







Geometric Series

• Geometric Series • Geometric Series - the sum of the terms of a geometric sequence.

• Geo. Sequence: 1, 3, 9, 27, 81

• Geo. Series: 1+3 + 9 + 27 + 81

• What is the sum of the geometric series?

Geometric Series • 1 + 3 + 9 + 27 + 81 = 121 • The formula for the sum S_n of the first n terms of a geometric series is given by

$$S_{n} = \frac{a_{1} - a_{1}r^{n}}{1 - r} \text{ or } S_{n} = \frac{a_{1}(1 - r^{n})}{1 - r}$$

Geometric Series • Find $! - 3(2)^{n-1}$ You can actually do it two ways. Let's use the old way. Plug in the numbers 1 - 4 for n and add. $\cdot [-3(2)^{1-1}] + [-3(2)^{2-1}] + [-3(2)^{3-1}]$

 $[-3(2)^{4-1}] + [-3(2)^{4-1}]$

Geometric Series · [-3(1)] + [-3(2)] + [-3(4)] + [-3(8)] =-3 + -6 + -12 + -24 = -45 The other method is to use the sum of geometric series formula.

- $3(2)^{n-1}$ use $S_n = \frac{a_1(1 - r^n)}{1 - r}$ n = 1• $a_1 = -3$, r = 2, n = 4

4 **Geometric Series** - 3 (2)^{*n*-1} use $S_n = \frac{a_1(1 - r^n)}{1 - r}$ n = 1 $\cdot a_1 = -3, r = 2, n = 4$ $S_4 = \frac{-3(1-2^4)}{1-2}$

Geometric Series $S_4 = \frac{-3(1-2^4)}{-3(1-2^4)}$ _ ? $S_4 = \frac{-3(1-16)}{-1}$ $S_4 = \frac{-3(-15)}{-1} = \frac{45}{-1} = -$ 45